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Physics-Informed Neural Networks for Acoustic Wave Propagation*

Marek Ružička[a], **Martin Štancel**[a], **Miroslav Imrich**[a], **Dmytro Havrysh**[a]

[a]Technical university of Košice, Department of Computers and Informatics
marek.ruzicka@tuke.sk,
martin.stance@tuke.sk
miroslav.imrich@tuke.sk
dmytro.havrysh@student.tuke.sk

Abstract

Acoustic wave propagation plays a fundamental role in various scientific and engineering disciplines, including medical imaging, seismology, and acoustics. Traditional numerical methods such as the Finite Element Method (FEM) and Finite Difference Method (FDM) are widely used to model these waves [1, 2], but they often suffer from computational inefficiencies, especially for high-dimensional problems or complex geometries. This work explores the application of Physics-Informed Neural Networks (PINNs) as an alternative approach, leveraging deep learning to solve wave equations efficiently [3]. PINNs integrate physical laws directly into the neural network's loss function, enabling solutions that adhere to the governing differential equations. We present a comparative analysis of PINNs with traditional numerical solvers, highlighting advantages, limitations, and potential improvements. Our experiments demonstrate that PINNs can effectively model wave propagation with comparable accuracy while reducing computational cost in certain scenarios.

Problem 1 (Physics-Informed Neural Networks for Wave Propagation)**.** *The problem of modeling acoustic wave propagation is traditionally solved using numerical methods. However, these approaches can be computationally expensive, particularly*

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in complex environments where high-resolution simulations are required. Additionally, handling complex boundary conditions and material heterogeneities presents challenges for classical solvers. It has been already shown that the PINNs can to improve simulation efficiency and accuracy, reducing reliance on extensive mesh generation and computational resources for wave propagation [4]. Our research is analyzing specifically acoustic waves.

Definition 2. A PINN is a deep learning model that embeds physical laws, such as differential equations, into its loss function to enhance learning. This reduces dependence on large datasets while ensuring solutions comply with fundamental physics.

The 2D acoustic wave equation describes wave propagation in a homogeneous medium:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \tag{1}$$

where $u(x, y, t)$ is the wave field, and c is the wave speed.

Common boundary conditions include Dirichlet (u prescribed on $\partial\Omega$), Neumann (normal derivative prescribed), and absorbing (reducing reflections).

Theorem 3. *A properly trained PINN can approximate solutions to the 2D acoustic wave equation with accuracy comparable to numerical solvers. The optimization minimizes the loss function:*

$$\mathcal{L} = \mathcal{L}_{physics} + \mathcal{L}_{data} + \mathcal{L}_{boundary}, \tag{2}$$

where $\mathcal{L}_{physics}$ enforces wave constraints, $\mathcal{L}_{data}$ accounts for data errors, and $\mathcal{L}_{boundary}$ ensures boundary condition compliance.

For an ideal PINN:

$$\lim_{N \to \infty} ||u_{PINN}(x, y, t) - u_{true}(x, y, t)|| = 0, \tag{3}$$

where u_{true} is the exact wave solution, given adequate training and a well-defined loss function.

Remark 4. While PINNs show promising accuracy, they require careful hyperparameter tuning and sufficient training data for optimal performance. Additionally, training PINNs for complex wave phenomena can be computationally demanding due to the need for high-resolution temporal and spatial domain coverage. Improving network architectures, such as using adaptive activation functions and domain decomposition techniques, can enhance convergence rates and accuracy.

Example 5. We apply PINNs to 1D, 2D and even 3D acoustic wave simulation, comparing the results against FEM and FDM solutions. The simulation involves a circular wave source in a heterogeneous medium with absorbing boundary conditions. The PINN-based approach demonstrates lower computational cost in high-dimensional settings while maintaining a relative error below 5% compared to numerical solvers. Additionally, PINNs are evaluated for their robustness in handling irregular geometries, showcasing their potential for practical applications in real-world problems. A simple example of PINN and analytical solution of Eq. 1 in 2D space for a sinusoidal wave source is presented at Fig. 1.

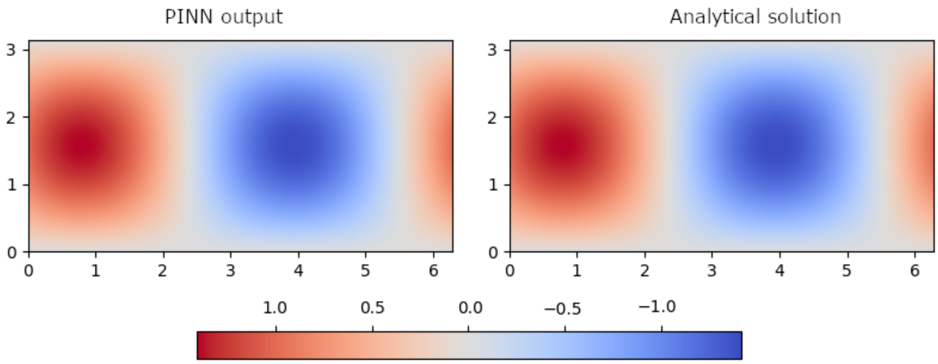

Figure 1. PINN and analytical output for 2D sinusoidal acoustic wave.

Corollary 6. *By integrating boundary conditions more effectively and optimizing network architectures, PINNs can further improve accuracy, making them a viable alternative to traditional solvers in specific scenarios. The hybridization of PINNs with traditional numerical methods, such as coupling PINNs with finite difference schemes for data augmentation, can lead to enhanced efficiency and broader applicability in wave simulation problems.*

References

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